Robustness Study of the SPRT for the Generalised Inverse Weibull Distribution when the underlying parameters are misspecified

Vaidehi Singh and Surinder Kumar
Department of Statistics, School for Physical and Decision Sciences,
Babasaheb Bhimrao Ambedkar University
(A Central University), Lucknow-226025, India.
e-mail: singhvaidehi92@gmail.com

Abstract

In this article, the problem of Sequential Probability Ratio Test (SPRT) is presented for generalised inverse Weibull distribution (GIWD). The GIWD has hazard function which has a unimodal shape. Hence, the GIWD could be an appropriate model for fitting the data which has the convex and then the concave shape. Robustness of the SPRT is studied for the parameters involved in the model, under the conditions when these parameters have undergone a change.

Keywords

Generalised inverse Weibull distribution, SPRT, OC and ASN functions, Robustness, Acceptance and rejection regions.

1 Introduction

Wald's (1947), idea of sequential testing procedure is motivated by the double sampling procedure of Dodge and Romig (1941), where the decision to draw the second sample or not, depends upon the observations of the first sample. They presented this scheme in recognition of the fact that they required, on an average, lesser number of sample observations than the single sampling procedure. Sequential testing procedures have been applied by several authors to handle different testing problems related to various probabilistic models. For a brief review of the literature, one may refer to Barnard, G. A. (1946 - 47), Oakland (1950), Epstein and Sobel (1955), Phatarford (1971), Foster et al. (1982), Fowler, G. W. (1983), Chaturvedi et al.

(2000). Robustness of the Sequential testing procedure in respect of OC and ASN functions, when the parameters under study are misspecified or undergone a change, is studied by several authors from time to time. For detailed review, one may refer to Montagne and Singpurwalla (1985), Hubbard et al. (1991), Chaturvedi et al. (1998), Surinder et al. (2018).

2 Set-up of the problem

Consider the cdf F(t) of the inverse Weibull distribution proposed by Drapella (1993)

$$F(t) = exp[-(\alpha/t)^{\beta}] \tag{2.1}$$

where t, α, $\beta > 0$. On upgrading F(t) to Z(t)$^{\gamma}$, the cdf given at (2.1) becomes $Z(t)^{\gamma} = exp[-\gamma(\alpha/t)^{\beta}]$ where $t, \alpha, \beta, \gamma > 0$ and the corresponding pdf is

$$z(t) = \gamma\beta\alpha^{\beta} exp[-\gamma(\alpha/t)^{\beta}], t > 0 \tag{2.2}$$

Equation (2.2) is the probability density function of generalised inverse Weibull distribution GIWD (α, β, γ), Felipe et al. (2011).

3 SPRT for testing the hypothesis regarding α

The Sequential Probability Ratio Test of strength $(\hat{\alpha}, \hat{\beta})$ for testing the simple null hypothesis $H_0 : \alpha = \alpha_0$ against the simple alternative $H_1 : \alpha = \alpha_1 (> \alpha_0)$ is as follows:
Let $x_i (i = 1, 2, 3, ...)$ be the successive observations on X. Now computing Z_i which is defined as

$$Z_i = \log \left[\frac{f(x_i; \alpha_1, \beta, \gamma)}{f(x_i; \alpha_0, \beta, \gamma)} \right] \tag{3.1}$$

$$Z_i = \beta \log \left(\frac{\alpha_1}{\alpha_0} \right) - \gamma(\alpha_1^{\beta} - \alpha_0^{\beta}) \left(\frac{1}{x_i} \right)^{\beta} \tag{3.2}$$

Choosing two constants A and B such that $0 < B < 1 < A$. The constant A and B are to be determined so that the test will have the prescribed strength $(\hat{\alpha}, \hat{\beta})$. For the purpose of practical computation, A and B are approximately given by,

$$A = (1 - \hat{\beta})/\hat{\alpha} \quad \text{and} \quad B = \hat{\beta}/(1 - \hat{\alpha}) \tag{3.3}$$

At the n^{th} stage, the process of taking observations is terminated with the acceptance (rejection) of H_0 if $\sum_{i=1}^{n} z_i \leq \log B$, (if $\sum_{i=1}^{n} z_i \geq \log A$) and if $\log B < \sum_{i=1}^{n} z_i < \log A$, the process is continued by taking an additional observation. The OC function of the SPRT for testing $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha = \alpha_1 (> \alpha_0)$ is given by,

$$L(\alpha) = \left[\frac{A^{h(\alpha)} - 1}{A^{h(\alpha)} - B^{h(\alpha)}} \right] \tag{3.4}$$

2

where A and B have been defined in equation (3.2) and for each value of α, the value of $h(\alpha)$ is to be determined, such that $h(\alpha) \neq 0$ and $E[e^{hZ}] = 1$

or

$$E\left[\frac{f(x;\alpha_1,\beta,\gamma)}{f(x;\alpha_0,\beta,\gamma)}\right]^h = 1 \tag{3.5}$$

$$\alpha^\beta\left[\left(\frac{\alpha_1}{\alpha_0}\right)^{\beta h} - 1\right] = h(\alpha_1^\beta - \alpha_0^\beta) \tag{3.6}$$

On solving the equation (3.6) by using the exponential function of the form $y = f(x) = a^x$ where $a > 1$, and retaining the terms upto 3rd degree, we get

$$\frac{\alpha^\beta h^2\beta^3}{3!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^3 + \frac{\alpha^\beta h\beta^2}{2!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^2 + \alpha^\beta\beta\log\frac{\alpha_1}{\alpha_0} - (\alpha_1^\beta - \alpha_0^\beta) = 0 \tag{3.7}$$

equation (3.7) is a quadratic equation in h. From this equation, we can compute h for $\alpha > 0$. The values of OC function $L(\alpha)$ may be obtained by using equation (3.3).
The ASN Function of the SPRT is given by

$$E[N|\alpha] = \frac{L(\alpha)\log B + [1 - L(\alpha)]\log A}{E(z)} \tag{3.8}$$

where,

$$E(Z) = \log\left(\frac{\alpha_1}{\alpha_0}\right)^\beta - \left[\frac{(\alpha_1^\beta - \alpha_0^\beta)}{\alpha^\beta}\right] \tag{3.9}$$

Thus, finally the ASN Function is given by

$$E[N|\alpha] = \frac{L(\alpha)\log B + [1 - L(\alpha)]\log A}{\log\left(\frac{\alpha_1}{\alpha_0}\right)^\beta - \left[\frac{(\alpha_1^\beta - \alpha_0^\beta)}{\alpha^\beta}\right]} \tag{3.10}$$

4 Robustness of the SPRT for the Parameter α

Let us suppose that the parameter 'γ' misspecified and has undergone a change then the pdf (2.3) becomes $f(x;\alpha,\beta,\gamma^*)$. The robustness of the SPRT presented in Section 3 with respect to OC and ASN functions is studied by obtaining the values of 'h' from the following equation

$$E_{\gamma^*}[e^{Zh}] = \int_0^\infty \left[\frac{f(x;\alpha_1,\beta,\gamma)}{f(x;\alpha_0,\beta,\gamma)}\right]^h f(x;\alpha,\beta,\gamma^*)\mathrm{d}x \tag{4.1}$$

3

$$\gamma^{*}\alpha^{\beta}\left[\left(\frac{\alpha_1}{\alpha_0}\right)^{\beta h}-1\right]=\gamma h(\alpha_1^{\beta}-\alpha_0^{\beta}) \tag{4.2}$$

Again, using the exponential function $y = f(x) = a^x$, $a > 1$ in equation (4.2) and retaining the terms upto 3rd degree in h, we get the following quadratic equation in h

$$\frac{h^3\beta^3}{3!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^3+\frac{\alpha^{\beta}h\beta^2}{2!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^2+(\alpha_1^{\beta}-\alpha_0^{\beta})\left(\frac{\gamma}{\gamma^*}\right)-\alpha^{\beta}\beta\log\frac{\alpha_1}{\alpha_0}=0 \tag{4.3}$$

$$\frac{h^3\beta^3}{3!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^3+\frac{\alpha^{\beta}h\beta^2}{2!}\left[\log\frac{\alpha_1}{\alpha_0}\right]^2+(\alpha_1^{\beta}-\alpha_0^{\beta})M-\alpha^{\beta}\beta\log\frac{\alpha_1}{\alpha_0}=0 \tag{4.4}$$

where $M = \dfrac{\gamma}{\gamma^*}$

Hence, the values of OC function $L(\alpha)$ is obtained from equation (3.3), on using the values of h for $\alpha > 0$ computed from equation (4.3). The robustness of the SPRT for the parameter α is analysed by considering the cases (i) $\gamma > \gamma^*$, (ii) $\gamma = \gamma^*$ and (iii) $\gamma < \gamma*$. The Robustness of the SPRT with respect to ASN function is studied by replacing the denominator in equation (3.7) by

$$E_{\gamma^*}(Z)=\log\left(\frac{\alpha_1}{\alpha_0}\right)^{\beta}-\frac{(\alpha_1^{\beta}-\alpha_0^{\beta})}{\alpha^{\beta}}\left(\frac{\gamma}{\gamma^*}\right) \tag{4.5}$$

After computing the values of ASN function under the cases $\gamma > \gamma^*, \gamma = \gamma^*$ and $\gamma < \gamma^*$, respectively, the robustness is studied accordingly.

5 SPRT for testing the hypothesis regarding γ

The SPRT for testing the null hypothesis $H_0 : \gamma = \gamma_0$ against the simple alternative $H_1 : \gamma = \gamma_1 (> \gamma_0)$ is defined as

$$Z_i=\log\frac{f(x_i;\alpha,\beta,\gamma_1)}{f(x_i;\alpha,\beta,\gamma_0)} \tag{5.1}$$

$$Z_i=\log\left(\frac{\gamma_1}{\gamma_0}\right)-\left(\frac{\alpha}{x_i}\right)^{\beta}(\gamma_1-\gamma_0) \tag{5.2}$$

The OC function of the SPRT for testing $H_o : \gamma = \gamma_o$ against , $H_1 : \gamma = \gamma_1 (> \gamma_0)$ is obtained by using the equation (3.3)

For each value of γ, the value of $h(\gamma)$ is to be determined, such that $h(\gamma) \neq 0$ and $\mathrm{E}\left[e^{hZ}\right] = 1$

or

$$E\left[\frac{f(x;\alpha,\beta,\gamma_1)}{f(x;\alpha,\beta,\gamma_0)}\right]^h=1 \tag{5.3}$$

4

$$\gamma\left\{\left[\frac{\gamma_1}{\gamma_0}\right]^h-1\right\}=h(\gamma_1-\gamma_o) \tag{5.4}$$

On solving the equation (5.4) by using the exponential function of the form $y=f(x)=a^x$ where $a>1$, and retaining the terms upto 3rd degree in h, we get

$$\frac{h^3}{3!}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^3+\frac{h}{2!}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^2+\log\left[\frac{\gamma_1}{\gamma_0}\right]-\frac{(\gamma_1-\gamma_0)}{\gamma}=0 \tag{5.5}$$

Hence, we obtain the quadratic equation in h. The ASN function of the SPRT is given by:

$$E\left[N|\gamma\right]=\frac{L(\gamma)\log B+[1-L(\gamma)]\log A}{E(Z)} \tag{5.6}$$

where,

$$E(Z)=\log\left[\frac{\gamma_1}{\gamma_0}\right]-\frac{\gamma_1-\gamma_0}{\gamma} \tag{5.7}$$

By putting the value of E(Z) in equation (5.5), we get the ASN as follows

$$E\left[N|\gamma\right]=\frac{L(\gamma)\log B+[1-L(\gamma)]\log A}{\left[\log\left[\frac{\gamma_1}{\gamma_0}\right]-\frac{\gamma_1-\gamma_0}{\gamma}\right]} \tag{5.8}$$

6 Robustness of the SPRT for Parameter γ

Let us suppose that the parameter α has undergone a change, then the pdf (2.3) becomes $f(x;\alpha^*,\beta,\gamma)$. Considering the equation

$$E_{\alpha^*}\left[e^{Zh}\right]=1$$

we have,

$$E_{\alpha^*}\left[e^{Zh}\right]=\int_0^\infty\left[\frac{f(x_i;\alpha,\beta,\gamma_1)}{f(x_i;\alpha,\beta,\gamma_0)}\right]^h f(x;\alpha^*,\beta,\gamma)dx \tag{6.1}$$

$$(\gamma_1-\gamma_0)\alpha^\beta h=\left\{\left[\frac{\gamma_1}{\gamma_0}\right]^h-1\right\}\gamma\alpha^{*\beta} \tag{6.2}$$

Again, using the exponential function $y=f(x)=a^x$, $a>1$ in equation (6.2) and retaining the terms upto 3rd degree in h, we get the following quadratic equation in h

$$\log\left[\frac{\gamma_1}{\gamma_0}\right]+\frac{h}{2}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^2+\frac{h^2}{3!}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^3=\left[\frac{\alpha}{\alpha^*}\right]^\beta\frac{(\gamma_1-\gamma_0)}{\gamma}$$

5

$$\frac{h^2}{3!}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^3+\frac{h}{2}\left\{\log\left[\frac{\gamma_1}{\gamma_0}\right]\right\}^2+\log\left[\frac{\gamma_1}{\gamma_0}\right]-N^{\beta}\frac{(\gamma_1-\gamma_0)}{\gamma}=0 \tag{6.3}$$

where $N=\dfrac{\alpha}{\alpha^*}$

The Robustness of SPRT with respect to ASN is studied by replacing the denominator of equation (5.6) by

$$E_{\alpha^*}[Z]=\int_0^\infty Zf(x;\alpha^*,\beta,\gamma)dx$$

On solving the above integral, we get

$$E_{\alpha^*}[Z]=\log\left(\frac{\gamma_1}{\gamma_0}\right)-\frac{(\gamma_1-\gamma_0)}{\gamma}\left(\frac{\alpha}{\alpha^*}\right)^{\beta} \tag{6.4}$$

Now, the values of ASN function is computed from equation (5.6) through using equation (6.4). The robustness of the SPRT for the parameter γ is studied by taking the cases, where (i) $\alpha>\alpha^*$, (ii) $\alpha=\alpha^*$ and (iii) $\alpha<\alpha*$ and then analyse the ASN curve to check the robustness.

7 Acceptance and Rejection boundaries for GIWD

We wish to test the simple hypotheses $H_0:\alpha=\alpha_0$ against $H_1:\alpha=\alpha_1$ having pre-assigned $0<\alpha,\beta,<1$. Let $A\approx\dfrac{1-\beta}{\alpha}$ and $B\approx\dfrac{\beta}{1-\alpha}$ and is defined as

$$Z_i=\beta\log\left(\frac{\alpha_1}{\alpha_0}\right)-\gamma(\alpha_1^{\beta}-\alpha_0^{\beta})\left(\frac{1}{x_i}\right)^{\beta} \tag{7.1}$$

Let us define, $Y(n)=\sum_{i=1}^{n}X_i$ and N=first integer $n(\geq 1)$, for which the inequality $Y(n)\leq c_1+dn$ or $Y(n)\geq c_2+dn$ holds with the constants

$$c_1=\frac{\ln B}{\gamma(\alpha_1-\alpha_0)},c_2=\frac{\ln A}{\gamma(\alpha_1-\alpha_0)}\text{ and }d=\frac{\ln\left(\frac{\alpha_1}{\alpha_0}\right)}{\gamma(\alpha_1-\alpha_0)} \tag{7.2}$$

8 Results and Conclusions

In order to study the robustness of the SPRT in respect of the OC and ASN functions, a simulation study is carried out. The robust behaviour of the parameters are studied through obtaining the numerical values and finally presented through graphs.

The theoretical expressions for the OC and ASN functions are obtained in Section 3 and 4 for the scale parameter α. The problem of testing simple null hypothesis versus simple

6

alternative hypothesis is considered by fixing $\alpha_0 = 15$ and $\alpha_1 = 17$, $\alpha = \beta = 0.05$. For varying values of γ, the numerical values of OC and ASN functions are obtained. Table : 8.1 and (Figure : 8.1 (a,b)) depicts the values (curves) for the OC and ASN functions for the parameter α. It is interested to point out that the numerical values (curves) obtained for M=1, $M > 1$ and $M < 1$ (see Table : 8.2 (a, b)) and Figures : 8.2 (a, b) shows that the SPRT is highly robust for a little misspecification in the parameter γ. The graphical representation of the OC and ASN function clarify that curves deviate towards right (left) for $M > 1$ ($M < 1$) from M=1. Thus, in case of parameter α involved in the model (2.3) the SPRT is highly sensitive.

Following the same procedure, the robust behaviour of the SPRT developed for γ is studied. The values (curves) of the OC and ASN functions for N=1, $N > 1$ and $N < 1$ are given in Table : 8.5 and Figure : 8.5 (a, b), Table : 8.6 (a, b) and Figure : 8.6 (a, b), respectively. Here, we observe that the OC and ASN curves shift towards the right (left) for $N > 1 (N < 1)$. It is evident from the observations that the SPRT is highly sensitive for even a minor change in the parameter α.

Alongwith, the study of scale parameters, the focus is also given to the shape parameter β, to study its effect, we considered different values of β i.e $\beta = 1, > 1$ and < 1 and the results are presented in Table : 8.3 (a, b), 8.4 (a, b), 8.7 (a, b), 8.8 (a, b); Figure : 8.3 (a, b), 8.4 (a, b), 8.7 (a, b), 8.8 (a, b). From this study, it is concluded that due to little variation in the parameter β there is a drastic change in the shape of the curves.

In Section 7, the problem of constructing the acceptance and rejection regions for H_0 under the case when $H_0 : \alpha_0 = 15$ vs $H_1 : \alpha_1 = 17$ for $\alpha = \beta = 0.05$ and $\gamma = 0.5$ is considered and the findings are presented in Figure (8.9). The obtained values of constants are $c_1 = -2.9444$, $c_2 = 2.9444$ and $d = 0.1431$, respectively. Finally, it is concluded that if $Y(N) \leq 0.1252N + 2.9444$, accept H_0 and if $Y(N) \geq 0.1252N - 2.9444$, accept H_1. At the intermediate stages, continue sampling.

7

9 Tables and Figures

Table 8.1 : OC and ASN Function for $(H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05)$		
α	$L(\alpha)$	$E(N)$
13.5	0.9996	127.9958
14.0	0.9978	165.6792
14.5	0.9891	225.6010
15.0	0.9502	324.4736
15.5	0.8062	466.0721
16.0	0.4847	553.7270
16.5	0.1798	477.2827
17.0	0.0498	352.7286
17.5	0.0126	263.8483
18.0	0.0032	208.2099
18.5	0.0008	172.3676
19.0	0.0002	147.9020

Table 8.2(a): OC Function for $\beta = 1$ $(H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05)$					
α	$M = 0.96$	$M = 0.98$	$M = 1$	$M = 1.02$	$M = 1.04$
13.5	0.9999	0.9998	0.9996	0.9990	0.9973
14.0	0.9996	0.9991	0.9978	0.9945	0.9859
14.5	0.9982	0.9956	0.9891	0.9729	0.9326
15.0	0.9915	0.9794	0.9502	0.8820	0.7409
15.5	0.9625	0.9127	0.8062	0.6180	0.3801
16.0	0.8549	0.7043	0.4847	0.2659	0.1195
16.5	0.5832	0.3592	0.1798	0.0770	0.0299
17.0	0.2549	0.1195	0.0498	0.0193	0.0071
17.5	0.0790	0.0325	0.0126	0.0047	0.0017
18.0	0.0215	0.0084	0.0032	0.0012	0.0004
18.5	0.0057	0.0022	0.0008	0.0003	0.0001
19.0	0.0015	0.0006	0.0002	0.0001	0.0000

8

Table 8.2(b): ASN Function for $\beta = 1$
$(H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05)$

α	$M = 0.96$	$M = 0.98$	$M = 1$	$M = 1.02$	$M = 1.04$
13.5	101.8310	113.4364	127.9958	146.7516	171.6749
14.0	125.6941	143.0194	165.6792	196.2657	238.8558
14.5	160.4444	187.9681	225.6010	278.2288	351.3720
15.0	214.3299	260.5186	324.4736	408.7443	500.1097
15.5	301.6078	376.7696	466.0721	539.3083	546.4275
16.0	432.1018	514.7887	553.7270	517.7341	434.0089
16.5	545.6783	542.9258	477.2827	390.7350	314.5998
17.0	513.6273	434.0089	352.7286	286.8392	237.4912
17.5	393.1464	320.4159	263.8483	221.5831	189.9508
18.0	293.2937	244.7105	208.2099	180.5099	159.0613
18.5	228.6541	196.8499	172.3676	153.1322	137.6969
19.0	187.1039	165.2808	147.9020	133.7875	122.1170

Table 8.3(a): OC Function for $\beta > 1$
$(H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05)$

α	$M = 0.98$	$M = 1$	$M = 1.02$
13.5	0.9990	0.9996	0.9998
14.0	0.9947	0.9979	0.9992
14.5	0.9741	0.9896	0.9958
15.0	0.8868	0.9523	0.9803
15.5	0.6292	0.8134	0.9162
16.0	0.2754	0.4964	0.7137
16.5	0.0805	0.1869	0.3700
17.0	0.0203	0.0521	0.1245
17.5	0.0049	0.0133	0.0341
18.0	0.0012	0.0033	0.0088

9

Table 8.3(b): ASN Function for $\beta > 1$ ($H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05$)			
α	$M = 0.98$	$M = 1$	$M = 1.02$
13.5	146.4394	127.7334	113.2132
14.0	195.9239	165.3690	142.7532
14.5	278.2979	225.3610	187.6875
15.0	412.8893	325.3565	260.5346
15.5	588.1994	476.0245	379.3985
16.0	285.7719	350.2118	427.2487
16.5	221.0826	262.9952	318.6569
17.0	180.1645	207.7392	243.9831
17.5	152.8516	172.0219	196.3978
18.0	133.5448	147.6163	164.9343

Table 8.4(a): OC Function for $\beta < 1$ ($H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05$)			
α	$M = 0.98$	$M = 1$	$M = 1.02$
13.5	0.9989	0.9996	0.9998
14.0	0.9942	0.9977	0.9991
14.5	0.9716	0.9886	0.9954
15.0	0.8770	0.9479	0.9785
15.5	0.6066	0.7988	0.9089
16.0	0.2565	0.4729	0.6946
16.5	0.0736	0.1728	0.3486
17.0	0.0184	0.0476	0.1146
17.5	0.0045	0.0120	0.0311
18.0	0.0011	0.0030	0.0080
18.5	0.0003	0.0008	0.0021
19.0	0.0001	0.0000	0.0005

Table 8.4(b): ASN Function for $\beta < 1$ $(H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05)$			
α	$M = 0.98$	$M = 1$	$M = 1.02$
13.5	147.0640	128.2587	113.6602
14.0	196.6034	165.9886	143.2857
14.5	278.1236	225.8304	188.2458
15.0	404.3683	323.5154	260.4790
15.5	489.2650	455.7250	373.9965
16.0	538.9801	980.1456	490.9139
16.5	394.5228	488.5765	585.9467
17.0	287.8846	355.1826	440.6188
17.5	222.0800	264.6894	322.1396
18.0	180.8551	208.6788	245.4315
18.5	153.4134	172.7137	197.3016
19.0	134.0308	148.1883	165.6279

Table 8.5 :OC and ASN Function for $\beta = 1$ $(H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05)$		
γ	$L(\gamma)$	$E(N)$
13.5	0.9995	127.9957
14.0	0.9978	165.6791
14.5	0.9891	225.6010
15.0	0.9501	324.4736
15.5	0.8062	466.0720
16.0	0.4846	553.7269
16.5	0.1797	477.2827
17.0	0.0498	352.7285
17.5	0.0126	263.8483
18.0	0.0031	208.2098
18.5	0.0007	172.3676
19.0	0.0002	147.9020

Table 8.6(a) : OC Function for $\beta = 1$ ($H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05$)					
α	$N = 1.04$	$N = 1.02$	$N = 1$	$N = 0.98$	$N = 0.96$
13.5	0.9999	0.9998	0.9995	0.999	0.9973
14.0	0.9996	0.9991	0.9978	0.9945	0.9859
14.5	0.9981	0.9956	0.9891	0.9729	0.9326
15.0	0.9914	0.9794	0.9501	0.8820	0.7409
15.5	0.9625	0.9127	0.8062	0.6180	0.3801
16.0	0.8549	0.7043	0.4846	0.2659	0.1195
16.5	0.5831	0.3592	0.1797	0.0770	0.0299
17.0	0.2548	0.1195	0.0498	0.0193	0.0071
17.5	0.0790	0.0325	0.0126	0.0047	0.0017
18.0	0.0214	0.0084	0.0031	0.0012	0.0004
18.5	0.0056	0.0022	0.0007	0.0003	0.0001
19.0	0.0015	0.0006	0.0002	0.0001	0.0000

Table 8.6(b): ASN Function for $\beta = 1$ ($H_0 : \alpha_0 = 15, H_1 : \alpha_1 = 17, \alpha = \beta = 0.05$)					
α	$N = 1.04$	$N = 1.02$	$N = 1$	$N = 0.98$	$N = 0.96$
13.5	101.8309	113.4364	127.9957	146.7516	171.6749
14.0	125.6941	143.0194	165.6791	196.2657	238.8558
14.5	160.4443	187.9681	225.6010	278.2288	351.3720
15.0	214.3298	260.5186	324.4736	408.7443	500.1096
15.5	301.6077	376.7697	466.0720	539.3083	546.4274
16.0	432.1017	514.7887	553.7269	517.7341	434.0090
16.5	545.6783	542.9258	477.2827	390.7350	314.5998
17.0	513.6273	434.0089	352.7285	286.8392	237.4912
17.5	393.1463	320.4159	263.8483	221.5831	189.9508
18.0	293.2937	244.7105	208.2098	180.5099	159.0613
18.5	228.6541	196.8499	172.3676	153.1322	137.6969
19.0	187.1039	165.2808	147.9020	133.7875	122.1170

12

Table 8.7(a) : OC Function for $\beta > 1$ ($H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05$)			
α	$N = 1.02$	$N = 1$	$M = 0.98$
13.5	0.9998	0.9996	0.9990
14.0	0.9991	0.9978	0.9945
14.5	0.9956	0.9891	0.9729
15.0	0.9794	0.9502	0.8819
15.5	0.9127	0.8062	0.6178
16.0	0.7045	0.4847	0.2657
16.5	0.3595	0.1798	0.0769
17.0	0.1196	0.0498	0.0193
17.5	0.0326	0.0126	0.0047
18.0	0.0084	0.0032	0.0011
18.5	0.0022	0.0008	0.0003
19.0	0.0006	0.0002	0.0001

Table 8.7(b) : ASN Function for $\beta > 1$ ($H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05$)			
α	$N = 1.02$	$N = 1$	$M = 0.98$
13.5	113.4233	127.9958	146.7728
14.0	142.9995	165.6792	196.3011
14.5	187.9359	225.6010	278.2902
15.0	260.4639	324.4736	408.8358
15.5	376.6843	466.0721	539.3553
16.0	514.7191	553.7270	517.6662
16.5	542.9654	477.2827	390.6516
17.0	434.0951	352.7286	286.7826
17.5	320.4813	263.8483	221.5471
18.0	244.7529	208.2099	180.4859
18.5	196.8779	172.3676	153.1153
19.0	165.3004	147.9020	133.7749

Table 8.8(a) : ASN Function for $\beta < 1$ $(H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05)$			
γ	$N = 1.02$	$N = 1$	$N = 0.98$
13.5	0.9998	0.9996	0.9990
14.0	0.9991	0.9978	0.9945
14.5	0.9956	0.9891	0.9729
15.0	0.9794	0.9502	0.8821
15.5	0.9126	0.8062	0.6182
16.0	0.7041	0.4847	0.2661
16.5	0.3590	0.1798	0.0771
17.0	0.1194	0.0498	0.0193
17.5	0.0325	0.0126	0.0047
18.0	0.0084	0.0032	0.0012
18.5	0.0022	0.0008	0.0003
19.0	0.0006	0.0002	0.0001

Table 8.8(b) : ASN Function for $\beta < 1$ $(H_0 : \gamma_0 = 15, H_1 : \gamma_1 = 17, \alpha = \beta = 0.05)$			
γ	$N = 1.02$	$N = 1$	$N = 0.98$
13.5	113.4494	127.9958	146.7304
14.0	143.0392	165.6792	196.2303
14.5	188.0003	225.6010	278.1673
15.0	260.5734	324.4736	408.6527
15.5	376.8550	466.0721	539.2613
16.0	514.8594	553.7270	517.8019
16.5	542.8852	477.2827	390.8183
17.0	433.9228	352.7286	286.8957
17.5	320.3505	263.8483	221.6191
18.0	244.6681	208.2099	180.5339
18.5	196.8219	172.3676	153.1492
19.0	165.2612	147.9020	133.8002

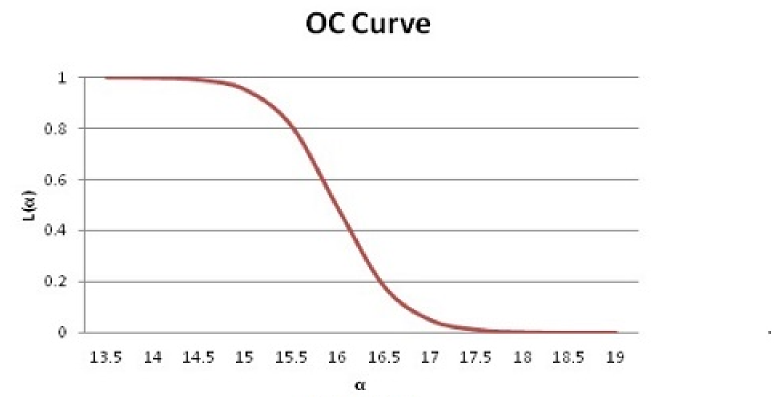

Fig: 8.1(a)

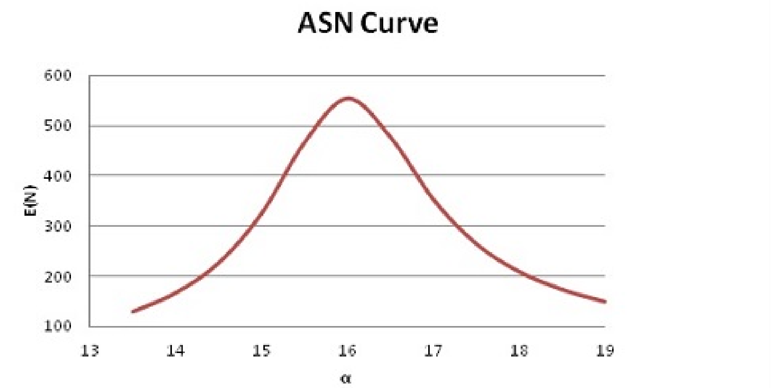

Fig: 8.1(b)

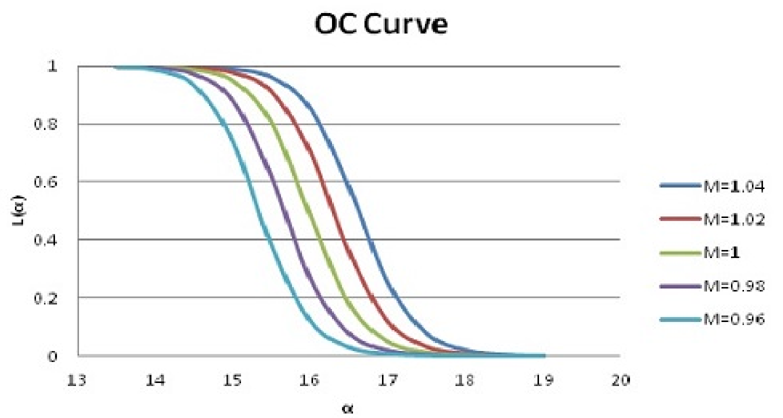

Fig: 8.2(a)

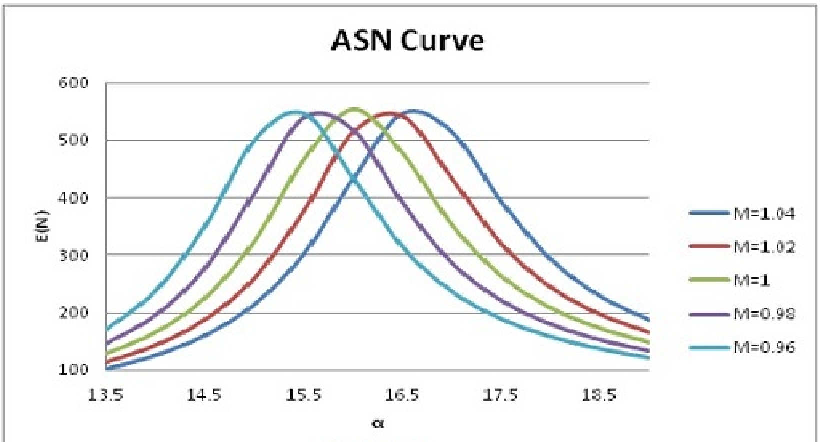

Fig: 8.2(b)

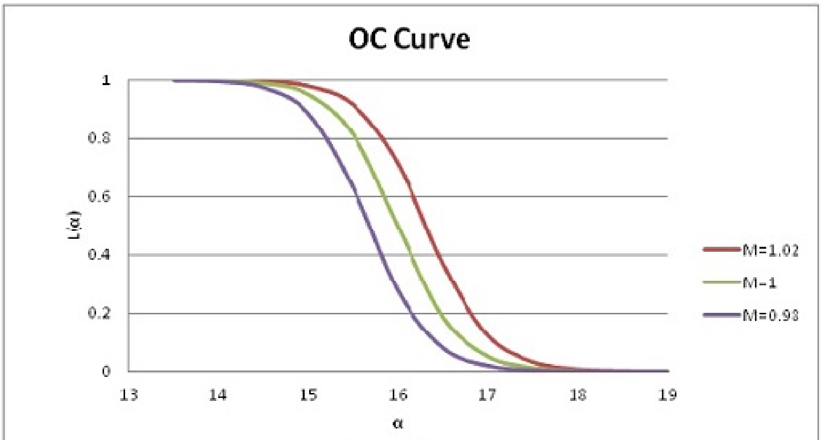

Fig: 8.3(a)

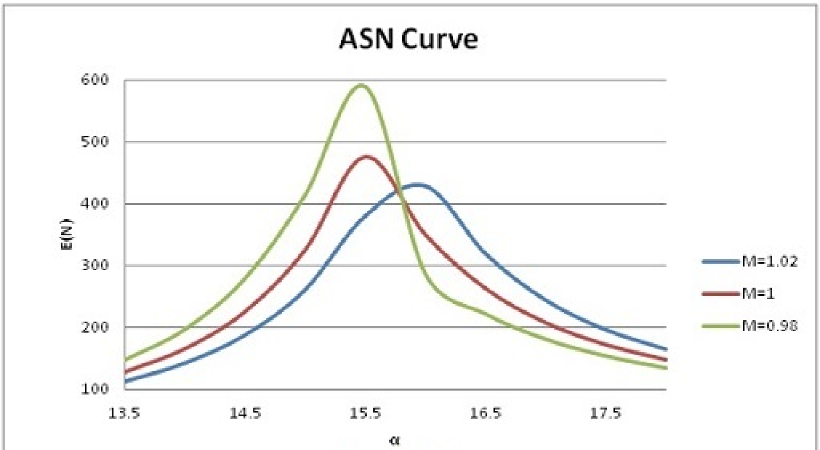

Fig: 8.3(b)

16

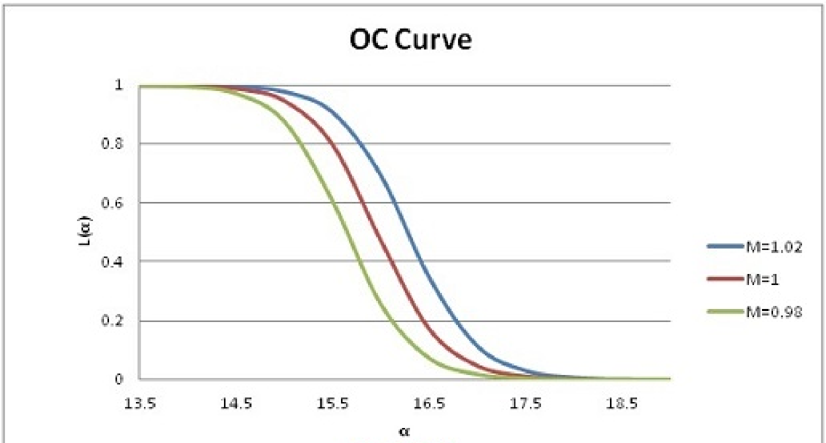

Fig: 8.4(a)

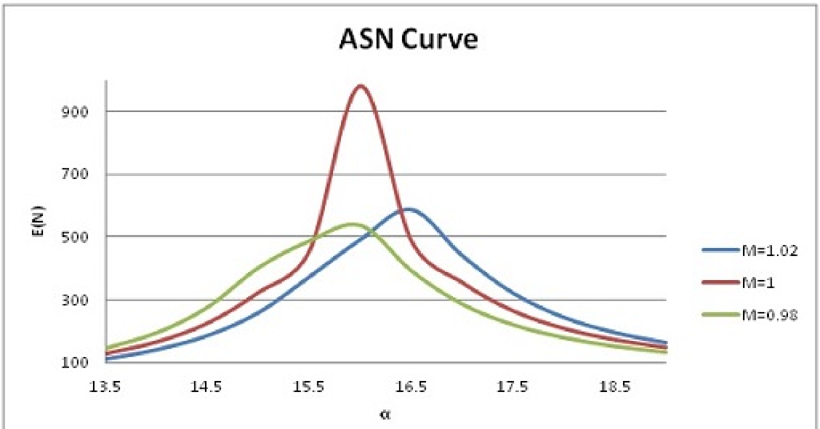

Fig: 8.4(b)

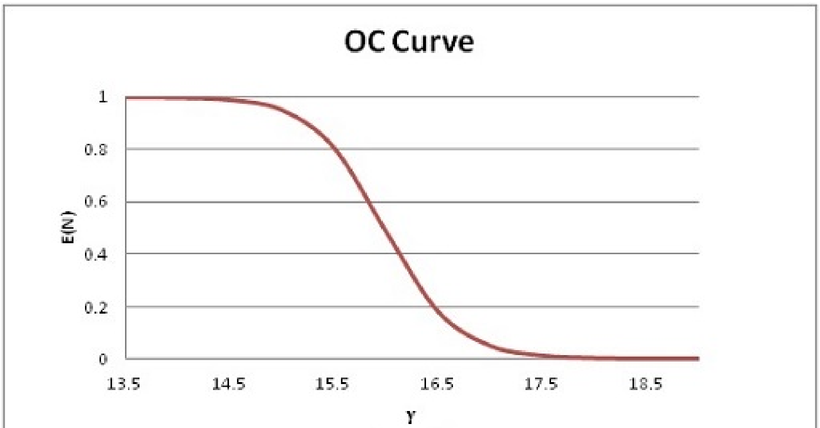

Fig: 8.5(a)

17

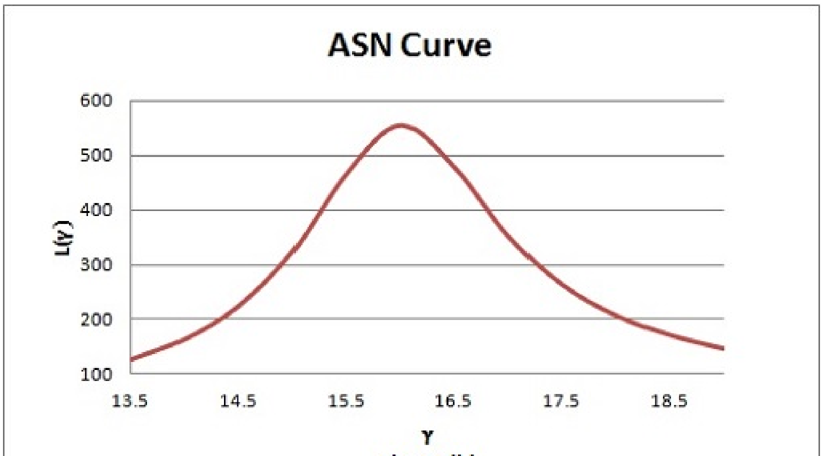

Fig: 8.5(b)

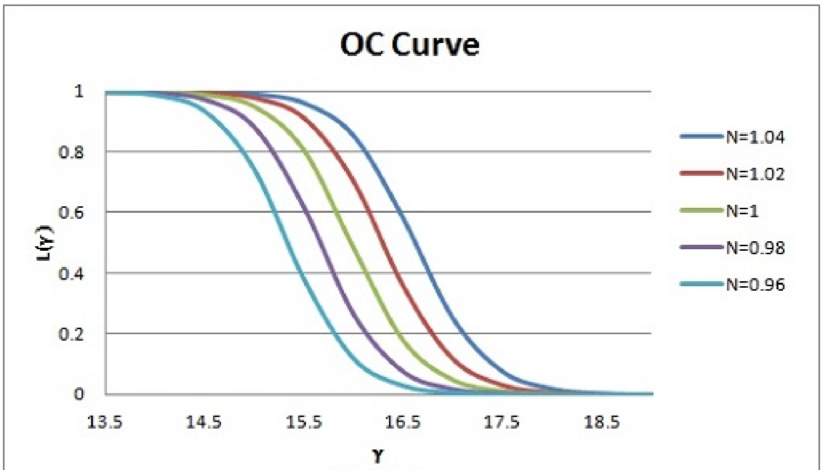

Fig: 8.6(a)

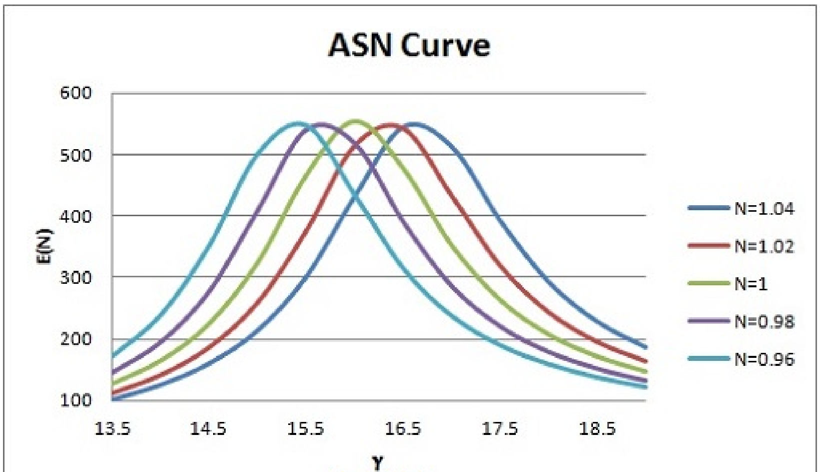

Fig: 8.6(b)

18

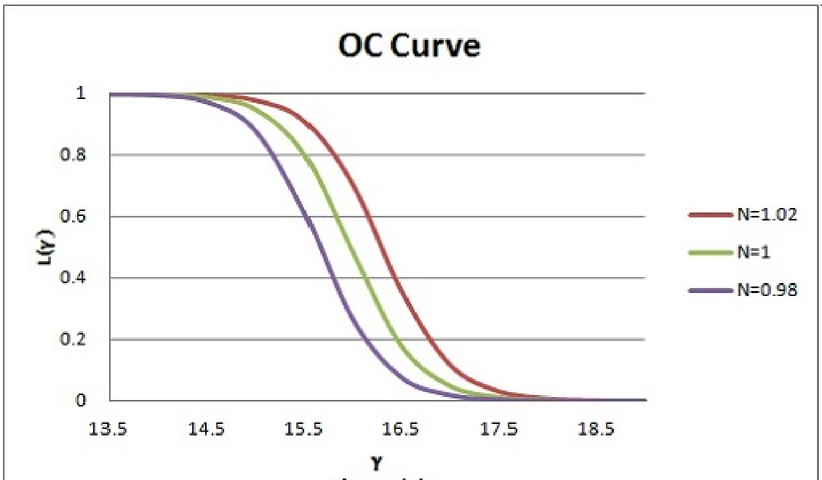

Fig: 8.7(a)

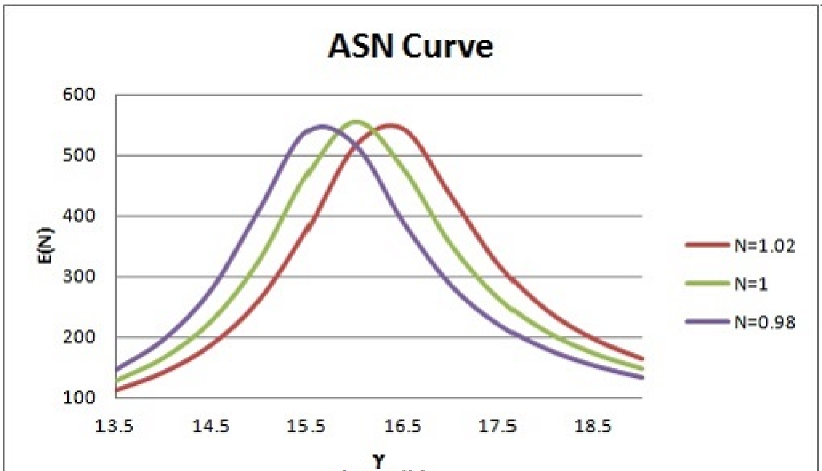

Fig: 8.7(b)

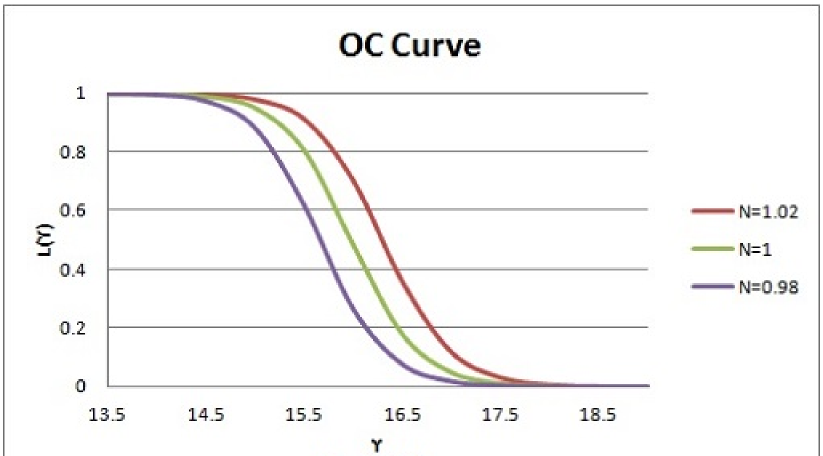

Fig: 8.8(a)

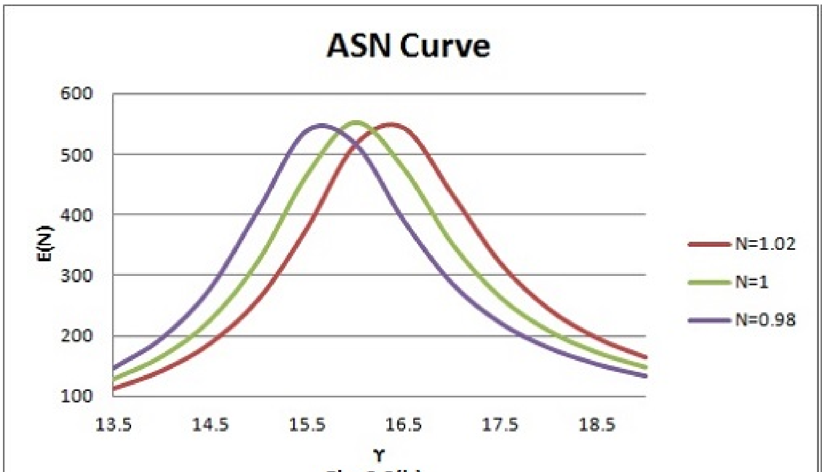

Fig: 8.8(b)

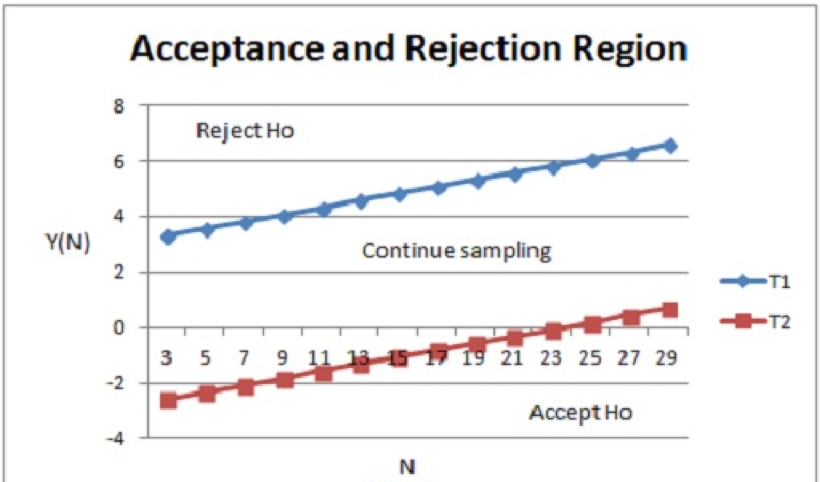

Fig: 8.9

20

References

[1] Barnard, G. A. (1946) : Sequential tests in industrial statistics. *J.Roy, Statist. Soc., Suppl.*, 8, 1 - 26.

[2] Barnard, G. A. (1947) : Review of A. Wald's sequential analysis. *J. Amer. Statist. Ass.*, 42, 658 - 664.

[3] Chaturvedi, A., Kumar, A. and Surinder, K. (1998) : Robustness of the sequential procedures for a family of life-testing models. *Metron*, 56, 117 - 137.

[4] Chaturvedi, A., Kumar, A. and Surinder, K. (2000) : Sequential testing procedures for a class of distributions representing various life testing models. *Statistical papers*, 41, 65 - 84.

[5] Drapella A. (1993) : Complementary Weibull distribution : unknown or just forgotten. *Qual Reliab Eng Int.* 9: 383 - 385.

[6] Dogde, H. F. and Romig, H. G. (1941) : Simple sampling and double sampling inspection tables. *Bell System Technical Journal*, 20, 1 - 61.

[7] Epstein, B. and Sobel, M. (1955) : Sequential life tests in exponential case. *Ann. Math. Statist.*, 26, 82 - 93.

[8] Felipe, Edwin and Gauss (2011) : The generalised inverse Weibull distribution. *Statistical papers*, 52(3), 591 - 619

[9] Foster. R. E., Tollefson, J. J. and Steffey, K. L. (1982) : Sequential sampling plan for adult corn rootworms. *Jour. Economic Entomology*, 75, 791 - 793.

[10] Fowler, G. W. (1983) : Accuracy of sequential sampling plans based on Wald's sequential probability ratio test, *Canadian Jour. Forest Res.*, 13, 1197 - 1203.

[11] Hubbard, D. J. and Allen, O. B. (1991) : Robustness of the SPRT for a negative binomial to misspecification of the dispersion parameter. *Biometrics*, 47, 419 - 427.

[12] Joshi, S. and Shah, M. (1990) : Sequential Analysis applied to testing the mean of an inverse Gaussian distribution with known coefficient of variation. Commun. Statist.-Theor. Meth., 19(4), 1457 - 1466.

[13] Montagne, E. R. and Singpurwalla, N. D. (1985) : Robustness of sequential exponential life-testing procedures. *Jour. American Statist. Assoc.*, 391, 715 - 719.

[14] Oakland, G. B. (1950) : An application of sequential analysis to whitefish sampling. *Biometrics*, 6, 59 - 67.

[15] Phatarfod, R. M. (1971) : A sequential test for Gamma distribution. *Jour. American Statist. Assoc.*, 66, 876 - 878.

[16] Surinder, K., Vaidehi, S. and Mayank, V. (2018) : Robustness study of the Sequential Testing Procedures for the New Weibull Pareto Distribution. *J. Stat. Appl. Pro. 7,* No. 1, 1 - 15.

[17] Wald, A. (1947) : Sequential Analysis. *John Wiley and Sons. New York.*