Effect of Inclined magnetic Field on the Flow of Jeffrey fluid through small diameters tubes

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Abstract
This paper deals with the effect of inclined magnetic field on the Jeffrey fluid flow through small diameters tubes. Considering the fluid non Newtonian, analytical expression for the velocity fluid has been derived and using it, a graphical study has been made for finding the effect of magnetic field on the flow profile for different angle of inclinations of tube. It has been found that the magnetic field has a great impact on velocity of fluid and also, it shows a resonance character around magnetic field (Bo) nearly equal 0.1.

Keywords: *Jeffrey Fluid, Magnetic Field, small diameter tube.*

Introduction

The study of magneto hydrodynamics flow problems has got considerable interest so for as its extensive applications in bio engineering and medical fields are concerned. It comprises the flow of electrically conducting fluids under the influence of magnetic field through various channels. The mutual interaction between the fluid motion and magnetic field is the necessary feature of any physical solution of such problems. Its theory is employed in the formation of pumps, heat exchangers, radar systems, flow meters, power generator etc. Some researchers have considered the MHD studies of different flow geometries of non-Newtonian and Newtonian fluids in many physical conditions. In this field, **Santhosh and Radhakrishnamacharya** [1] studied the effect of a magnetic field on a two-fluid model for the flow of a Jeffrey fluid in tubes of small diameters in the existence of a magnetic field. He obtained that the effect on velocity of Jeffrey parameter. **Pal, Misra and Gupta** [2] found an asymptotic series solution for steady flow of an incompressible, viscous and electrically conducting fluid in a channel permeated by a uniform transverse magnetic field such that the depth of channel varied slowly in the axial direction. Analytical expressions were derived for the pressure drop along the channel and the wall shear stress. It was obtained that for a fixed value of the Reynolds number the wall shear stress increases with increase in the magnetic parameter. Numerical computations were carried out for some specific slowly varying channels show that for fixed. **Aroesty and Gross** [3] derived primarily a mathematical analysis of the Casson fluid, which possessed finite yield stress and shear-dependent viscosity when it was subjected to a periodic pressure gradient in a long rigid tube. The coupled nonlinear implicit equations of motion and constitutive relations were non dimensional and approximate solutions valid for small values of the Womersley frequency parameter were derived. **Vajravelu**, **Sreenadh**, **and Lakshminarayana** [4] studied the peristaltic flow of a Jeffrey fluid in a vertical porous stratum with heat transfer under long wavelength and low Reynolds number assumptions. The nonlinear governing equations were solved using perturbation technique. The expressions for velocity, temperature and the pressure rise per one wave length were determined. The effects of different parameters on the velocity, the temperature and the pumping characteristics were discussed. It was observed that the effects of the Jeffrey number, Grashof number, the perturbation parameter, and the peristaltic wall deformation parameter were very effective. The results obtained for the flow and heat transfer characteristics revealed many interesting behaviors that led further study on the non-Newtonian fluid phenomena, especially the shear-thinning phenomena. Shear-thinning reduces the wall shear stress. **Jyothi**, **Devaki and Sreenadh** [5] examined the pulsatile flow of a Jeffrey fluid in a circular tube plated internally with porous material and found that the velocity, flux of the fluid flow and the concentration drag in the tube. **Kothandapani and Srinivas** [6] studied the peristaltic motion of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. The expressions for stream function of axial pressure gradient and axial velocity were obtained. **Hayat, Ali, Asghar and Siddiqui** [7] studied Peristaltic flow of a Jeffrey fluid through the gap between concentric uniform tubes with particular reference to an endoscope effects. The velocity components and axial pressure gradient were obtained analytically. Numerical integration was used to analyze the pressure rise and frictional forces on the inner and outer tubes.

Formulation of problem

Assuming that a laminar, steady and axis symmetric flow of an electrically conducting Jeffrey fluid is occurring through a rigid circular tube of radius **a** and an inclined magnetic field B_0 is applied to the flow, the flow in the tube is described as two-fluid model consisting of core region of radius **b**, occupied by Jeffrey fluid and peripheral region of portliness $\varepsilon = (a-b)$ overfull by Newtonian fluid, as shown in Figure 1. Let μ_p is viscosity of Newtonian fluid and μ_c is viscosity of Jeffrey fluid. Assuming cylindrical polar coordinate system (**R, θ,** ***Z***) in the z axis is taken along the axis of the tube. Here ***q*** is the velocity of the fluid, ***j*** is the current density of fluid, $B = (B_1 + B_0)$ is the whole magnetic field, $\boldsymbol{B_1}$ is the induced magnetic field and $\boldsymbol{j} \times \boldsymbol{B}$ is Lorentz's force which is the body force applying on the fluid, according Maxwell equations and by Ohm's law

$$\nabla \cdot B = 0 \quad \nabla \times B = \mu_m \quad \nabla \times E = -\frac{\partial B}{\partial T} \quad J = \sigma(E + q \times E) \qquad \ldots\ldots (1)$$

Where σ electrical conductivity, μ_m is the magnetic permeability and ***E*** is the electric field. The imposed and induced electric fields are assumed to be negligible.

Hence, the force

$$J \times B = -\sigma B_o^{\,2} w \qquad \ldots\ldots (2)$$

The constitutive equations for an incompressible Jeffery fluid

$$\bar{T} = -P\bar{I} + \bar{S}$$
$$\bar{S} = \frac{\mu_c}{1+\lambda_1}\left[\dot{\bar{\gamma}} + \lambda_2 \ddot{\bar{\gamma}}\right] \qquad \ldots\ldots (3)$$

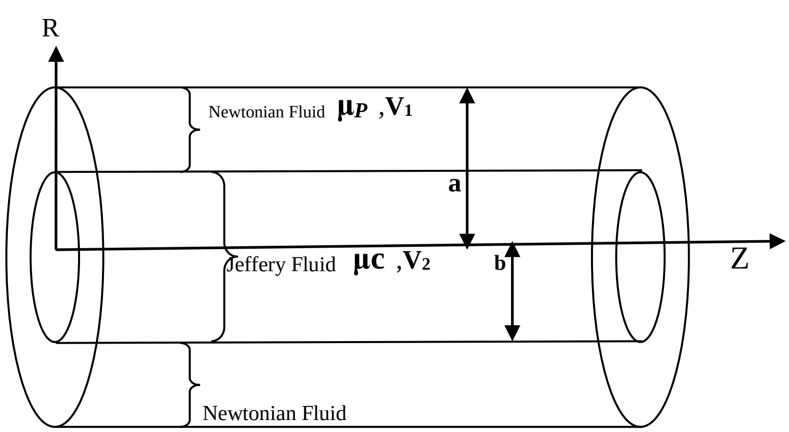


Figure 1

$\overline{T}$: Cauchy stress tensor

$\overline{S}$: Extra stress tensor

P: pressure of fluid

$\overline{I}$: Identity tensor

λ_1 : Ratio of relaxation to retardation times

λ_2 : Retardation time,

$\dot{\gamma}$: Shear rate and dots is denoted differentiation with respect to time.

Equation of continuity

$$\frac{\partial V_R}{\partial R} + \frac{V_R}{R} + \frac{\partial V_Z}{\partial Z} = 0 \qquad \text{...... (4)}$$

And

$$\rho\left[V_R \frac{\partial}{\partial R} + V_Z \frac{\partial}{\partial Z}\right]V_R = -\frac{\partial P}{\partial R} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\overline{S}_{RR}\right) + \frac{\partial}{\partial Z}\left(\overline{S}_{RZ}\right) \qquad \text{...... (5)}$$

$$\rho\left[V_R \frac{\partial}{\partial R} + V_Z \frac{\partial}{\partial Z}\right]V_Z = -\frac{\partial P}{\partial Z} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\overline{S}_{ZR}\right) + \frac{\partial}{\partial Z}\left(\overline{S}_{ZZ}\right) - \sigma B_O{}^2\, Sin^2\theta V_Z \qquad \text{...... (6)}$$

In which

$$\overline{S}_{RR} = \frac{2\mu_C}{1+\lambda_1}\left[1+\lambda_2\left(V_R \frac{\partial}{\partial R} + V_Z \frac{\partial}{\partial Z}\right)\right]\frac{\partial V_R}{\partial R} \qquad \text{...... (7)}$$

$$\overline{S}_{RZ} = \overline{S}_{ZR} = \frac{\mu_C}{1+\lambda_1}\left[1+\lambda_2\left(V_R \frac{\partial}{\partial R} + V_Z \frac{\partial}{\partial Z}\right)\right]\left(\frac{\partial V_Z}{\partial R} + \frac{\partial V_R}{\partial Z}\right) \qquad \text{...... (8)}$$

$$\overline{S}_{ZZ} = \frac{2\mu_C}{1+\lambda_1}\left[1+\lambda_2\left(V_R \frac{\partial}{\partial R} + V_Z \frac{\partial}{\partial Z}\right)\right]\left(\frac{\partial V_Z}{\partial Z}\right) \qquad \text{...... (9)}$$

Where V_R , V_Z are the velocity components in the **R** and **Z** directions respectively, **P** is the pressure of fluid , $\boldsymbol{\rho}$ is the density, $\bar{S}_{RR}$, $\bar{S}_{RZ}$, $\bar{S}_{ZR}$, $\bar{S}_{ZZ}$ are the extra stress components, θ is the angle of inclination and $M = \sqrt{\frac{\sigma}{\mu} B_o{}^2 a^2}$ is the magnetic parameter. Consider the flow is in **Z**-direction only and hence the velocity component $V_R = 0$. The governing equations of flow of Jeffrey fluid in the core region $(0 \le r \le b)$ reduce to

$$\frac{\partial P}{\partial R} = 0 \qquad \text{...... (10)}$$

$$\frac{\mu_C}{1+\lambda_1} \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial V_Z}{\partial R} \right) - \sigma B_o{}^2 \, Sin^2\theta V_Z - \frac{\partial P}{\partial Z} = 0 \qquad \text{...... (11)}$$

Let $V_Z(R) = V_1(R)$ be the velocity for peripheral region and $V_2(R)$ for core region. The governing equations flow of fluid are:

Peripheral region (Newtonian fluid)

$$\mu_P \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial V_1}{\partial R} \right) - \sigma B_o{}^2 \, Sin^2\theta V_1 - \frac{\partial P}{\partial Z} = 0 \qquad for\ b \le R \le a \qquad \text{...... (12)}$$

Core region (Jeffery fluid)

$$\frac{\mu_C}{1+\lambda_1} \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial V_2}{\partial R} \right) - \sigma B_o{}^2 \, Sin^2\theta V_2 - \frac{\partial P}{\partial Z} = 0 \qquad for\ b \le R \le a \qquad \text{...... (13)}$$

And $\frac{\partial P}{\partial Z}$ is constant pressure gradient. Using following boundary conditions

$$\begin{aligned} &V_1 = 0 \ \ at \ \ R = a \\ &V_1 = V_2, \quad \tau_1 = \tau_2, \quad R = b \\ &V_2 \, is \ \ finite \ \ at \ R = 0 \end{aligned} \qquad \text{...... (14)}$$

From equation (12), obtain

$$\mu_P \frac{1}{R}\left(\frac{\partial}{\partial R}\left(R\frac{\partial V_1}{\partial R}\right)\right) - \sigma B_o{}^2 Sin^2\theta V_1 - \frac{\partial P}{\partial Z} = 0 \qquad \ldots\ldots (15)$$

$$\mu_P\left(\frac{\partial^2 V_1}{\partial R^2} + \frac{1}{R}\frac{\partial V_1}{\partial R}\right) - \sigma B_o{}^2 Sin^2\theta V_1 - \frac{\partial P}{\partial Z} = 0 \qquad \ldots\ldots (16)$$

Now, introduce new transformation

$$Y = \frac{R}{R_o} \quad \Rightarrow \frac{\partial Y}{\partial R} = \frac{1}{R_o} \qquad \ldots\ldots (17)$$

$$\frac{\partial V_1}{\partial R} = \frac{\partial V_1}{\partial Y}.\frac{\partial Y}{\partial R} \qquad \ldots\ldots (18)$$

$$\frac{\partial V_1}{\partial R} = \frac{\partial V_1}{\partial Y}.\frac{1}{R_o} \qquad \because \quad \frac{\partial Y}{\partial R} = \frac{1}{R_o} \qquad \ldots\ldots (19)$$

$$\frac{\partial^2 V_1}{\partial R^2} = \frac{\partial^2 V_1}{\partial Y^2}.\frac{1}{R_o} \qquad \ldots\ldots (20)$$

From equation (16) using equations (17) to (20)

$$\mu_P\left(\frac{1}{R_o{}^2}\frac{\partial^2 V_1}{\partial Y^2} + \frac{1}{YR_o}\frac{\partial V_1}{\partial Y}\frac{1}{R_0}\right) - \sigma B_o{}^2 Sin^2\theta V_1 - \frac{\partial P}{\partial Z} = 0 \qquad \ldots\ldots (21)$$

$$\frac{\mu_P}{R_o{}^2}\left(\frac{\partial^2 V_1}{\partial Y^2} + \frac{1}{Y}\frac{\partial V_1}{\partial Y}\right) - \sigma B_o{}^2 Sin^2\theta V_1 - \frac{\partial P}{\partial Z} = 0 \qquad \ldots\ldots (22)$$

$$\left(\frac{\partial^2 V_1}{\partial Y^2} + \frac{1}{Y}\frac{\partial V_1}{\partial Y}\right) - \sigma B_o{}^2 Sin^2\theta \frac{R_o{}^2}{\mu_P} V_1 + \frac{R_o{}^2 P}{\mu_P} = 0 \qquad \ldots\ldots (23)$$

$$\because -\frac{\partial P}{\partial Z} = P$$

The above the differential equation (23) represents a Bessel's equation using boundary condition from equation (14) hence its solution can be given as:

$$V_1(R_O)=\frac{P}{\sigma B_o^{\,2}\,Sin^2\theta}\left[1-\frac{J_o\left(R_o\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}{J_o\left(a\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}\right] \qquad \text{...... (24)}$$

Similarly solve from equation (13) and (14) solution following

$$V_2(R_O)=\frac{P}{\sigma B_o^{\,2}\,Sin^2\theta}\left[2-\frac{J_o\left(b\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}{J_o\left(a\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}-\frac{J_o\left(R_o\sqrt{\dfrac{(1+\lambda_1)\sigma B_o^{\,2}\,Sin^2\theta}{\mu_c}}\right)}{J_o\left(b\sqrt{\dfrac{(1+\lambda_1)\sigma B_o^{\,2}\,Sin^2\theta}{\mu_c}}\right)}\right] \qquad \text{...... (25)}$$

The flow flux in the peripheral region, denoted by Q_P is defined by

$$Q_P=2\pi a^2\int_t^1 V_1(R_o)\;R_o\,dR_o \qquad \text{...... (26)}$$

Substituting for $V_1(R_o)$ from (24) in (26)

$$Q_P=\frac{P\pi\,a^2}{\sigma B_o^{\,2}\,Sin^2\theta}\left(1-t^2+2\frac{t\,J_1\left(t\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)-J_1\left(\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}{\left(\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)J_o\left(\sqrt{\dfrac{\sigma B_o^{\,2}\,Sin^2\theta}{\mu_P}}\right)}\right) \qquad \text{...... (27)}$$

Similarly, the flow flux in the core region is given by

$$Q_C=2\pi a^2\int_0^t V_1(R_o)\;R_o\,dR_o$$

$$Q_C = \frac{P\pi a^2}{\sigma B_o^{\ 2} Sin^2\theta}\left(2t^2 - t^2 \frac{J_0\left(t\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)}{J_o\left(\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)} - 2\frac{t\, J_1\left(t\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right)}{\left(\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right) J_o\left(t\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right)} \right)$$

Where $t=b/a$

Total flow of flux

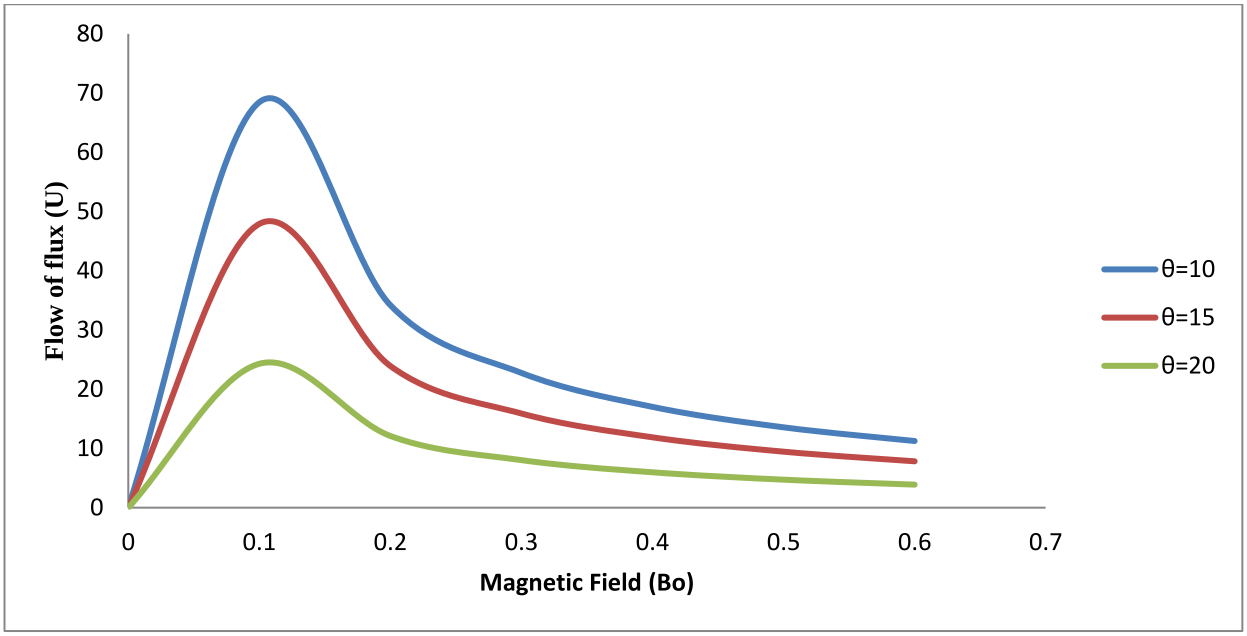
$Q = Q_P + Q_C$

$$Q = \frac{P\pi a^2}{\sigma B_o^{\ 2} Sin^2\theta}\left(\begin{array}{l} 1+t^2+2\frac{t\, J_1\left(t\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right) - J_1\left(\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)}{\left(\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right) J_o\left(\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)} - t^2 \frac{J_0\left(t\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)}{J_o\left(\sqrt{\frac{\sigma B_o^{\ 2} Sin^2\theta}{\mu_P}}\right)} \\ -2\frac{t\, J_1\left(t\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right)}{\left(\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right) J_o\left(t\sqrt{\frac{(1+\lambda_1)\sigma B_o^{\ 2} Sin^2\theta}{\mu_c}}\right)} \end{array} \right)$$

Results and Discussion

Sr. No.	Magnetic Field (Bo)	Flow of flux (Q)		
		θ=10	θ =15	θ =20
1	0	0	0	0
2	0.1	68.46035	47.90585	24.29299
3	0.2	34.19109	23.9138	12.10723
4	0.3	22.75046	15.8988	8.027356
5	0.4	17.01678	11.87779	5.973372

6	0.5	13.56565	9.45403	4.729063
7	0.6	11.25556	7.828541	3.888821



The above graph shows that the total flow of flux (Q) with respect to Magnetic Field (Bo) for different values of angle of inclination of magnetic field as magnetic field increases the total flux also increases from the value nearly equal to zero and takes maximum values around Bo=0.1,and then decreases sharply showing a resonance character . This is fully describing the physical nature of the problems. This value of magnetic field for which resonance occurs for different inclinations of plate can be used for controlling the and in turn, after proper study, results can be applied for industrial purpose in checking the moment of such fluids under the influence of Magnetic field.

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